5M/MTH–302 Syllabus–2023

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(Nov–Dec)

FYUP : 5th Semester Examination

MAJOR

MATHEMATICS

(Numerical Methods and Optimization Techniques)

MTH–302

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Compute the smallest positive root of $x^3 - 9x + 1 = 0$ by Bisection method, correct to two decimal places. 6

(b) Find a real root of the equation $x^4 - x - 10 = 0$, correct to three decimal places by Newton-Raphson method. 5

(c) Solve the following system by Gauss' elimination method : 7

$$2x+y+z=10$$
$$3x+2y+3z=18$$
$$x+4y+9z=16$$

2\. (a) Applying Newton's forward interpolation formula, find the cubic polynomial which takes the following values :

$$y(0)=1,\ y(1)=0,\ y(2)=1,\ y(3)=10$$

Hence or otherwise, obtain $y(4)$. 5+1=6

(b) Using Lagrange's interpolation formula, find the polynomial of degree two which attains the following tabular values : 6

x	1	2	4
y	–9	–12	–24

(c) Find the polynomial of the lowest possible degree buy applying Newton's divided difference formula from the following table : 6

x	3	2	1	–1
y	3	12	15	–21

UNIT—II

3\. (a) Find $\frac{dy}{dx}$ at $x=3{\cdot}0$ for the function $y=f(x)$ from the following table : 6

x	3·0	4·0	5·0	6·0
y	4·0552	4·9530	6·0496	7·3891

(b) Find the value of the integral

$$\int_0^1 \frac{dx}{1+x^2}$$

by using Simpson's $\frac{1}{3}$rd rule. Hence, obtain the approximate value of π. 6+1=7

(c) Find the approximate value of

$$\int_0^1 \frac{x\,dx}{1+x^2}$$

up to three decimal places by using Trapezoidal rule, taking 6 equal subintervals of [0, 1]. 6

4\. (a) Apply Euler's method to the initial value problem

$$\frac{dy}{dx}=x+y,\ y=0$$

when $x=0$ from $x=0$ to $x=1{\cdot}0$, taking $h=0{\cdot}2$. 7

(b) Using Runge-Kutta method of fourth order, solve

$$\frac{dy}{dx} = x + y^2 \text{ with } y(0) = 1$$

at $x = 0 \cdot 2$. Take $h = 0 \cdot 2$. 6

(c) Using Runge-Kutta method of order two, compute $y(0 \cdot 2)$ for the equation

$$\frac{dy}{dx} = \frac{y-x}{y+x}, \; y(0) = 1$$

Take $h = 0 \cdot 2$. 6

UNIT—III

5. *(a)* Explain the following : 3+1=4

(i) The characteristics of the canonical form of LPP

(ii) Slack and surplus variables

(b) Rewrite the following LPP in standard form : 5

Minimize $Z = 2x_1 + x_2 + 4x_3$

subject to the constraints

$$-2x_1 + 4x_2 \leq 4$$
$$x_1 + 2x_2 + x_3 \geq 5$$
$$2x_1 + 3x_3 \leq 2$$
$$x_1, x_2 \geq 0$$

and x_3 unrestricted in sign.

(c) Show that the following system of linear equations has a degenerate solution : 5

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

(d) Find all the basic feasible solutions of the following LPP : 5

Maximize $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$

subject to the constraints

$$2x_1 + 3x_2 + x_3 + 4x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

and choose the one which maximizes Z.

6. *(a)* Use simplex method to solve the following LPP : 13

Maximize $Z = 2x_1 + 3x_2$

subject to the constraints

$$x_1 + x_2 \leq 4$$
$$-x_1 + x_2 \leq 1$$
$$x_1 + 2x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

(b) Obtain the dual problem of the following LPP : 6

Maximize $Z = 2x_1 + 5x_2 + 6x_3$

subject to the constraints

$$5x_1 + 6x_2 - x_3 \le 3$$
$$-2x_1 + x_2 + 4x_3 \le 4$$
$$x_1 - 5x_2 + 3x_3 \le 1$$
$$-3x_1 - 3x_2 + 7x_3 \le 6$$
$$x_1, x_2, x_3 \ge 0$$

UNIT—IV

7. *(a)* Prove that the necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda$$

where a_i = quantity of commodity available at origin i, b_j = quantity of commodity needed at destination j. 5

(b) Determine an initial basic feasible solution of the following transportation problem using Vogel's approximation method : 10

Source	*Destination*				*Availability*
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Requirement	60	40	30	110	240

(c) Explain the following : 2+2=4

(i) Duality in transportation problem

(ii) Loops in transportation table

8. *(a)* With reference to a transportation problem, explain the following : 2+3=5

(i) Optimality test

(ii) Degeneracy

(b) Solve the following transportation problem : 14

Source	*Destination*				*Availability*
	1	2	3	4	
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Requirement	6	10	12	15	43

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